

A STUDY OF RADAR CLUTTER MODELS

By

MANDAVA RAJESWARI

TH
EE/1980/M
EE R1378

1980

M

RAJ

STU



DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

NOVEMBER, 1980

A STUDY OF RADAR CLUTTER MODELS

A Th S b m t t d
P t l F l f l m t f t h R q m t
f t h D g f
MASTER OF TECHNOLOGY

By
MANDAVA RAJESWARI

t th

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
NOVEMBER 1980

EE 1980 -M RAT- STU

1
CIN LIBRARY
acc. No. A 66955

7 SEP 1981

CERTIFICATE

This is to certify that the thesis entitled A STUDY OF RADAR CLUTTER MODELS by Sandeep Raj has been carried out under my supervision and has not been submitted elsewhere for a degree

Prakash

(P R K Rao)
Professor

Department of Electrical Engineering
Indian Institute of Technology
Kanpur 16

ACKNOWLEDGEMENTS

I wish to express my heartfelt gratitude to my thesis supervisor Dr P R K Rao who has been my mentor throughout my M Tech programme His encouragement and constructive criticism are greatly valued

I would like to thank Dr Ramesh Garg for his suggestions in the early stages of this thesis

I greatly acknowledge Miss T Pramila Dr D P Rao and family for making my stay at Kanpur memorable

I thank Mr B R K Reddy for his help during the various stages of this thesis

I finally thank Mr C M Abraham for his excellent typing

(MANDAVI RAJESWARI)

ABSTRACT

Clutter which degrades the probability of target detection performance of a radar is a term used to denote the composite echoes from unwanted targets illuminated by the radar beam. Clutter return unlike additive thermal noise depend on radar's own transmission. Therefore mere increase in the transmitted power without a suitably chosen signalling waveform does not necessarily result in improved target detection performance. Considerations of optimum waveform and receiver designs presume a knowledge of the statistical description of clutter returns. Specifications of the scattering function which provides a measure of the distribution of clutter power in delay and doppler variables is often considered adequate.

This thesis deals with the evaluation of the scattering function for two models of clutter. In the first the scatterers are modelled as random rotating dipoles with an overall drift velocity and differential velocities. An expression for the scattering function is derived by calculating the voltage reflection coefficients of an ensemble of dipoles with non homogeneous Poisson distribution. An attempt has been made to generate a few scattering functions which compare favourably with some of the reported experimental results.

In the second approach clutter targets are modelled as a collection of ellipsoidal scatterers. The ellipsoids are assumed to fluctuate in size and location about their mean positions. As an illustration an expression for the scattering function is obtained for a relatively simple geometry and aspect angle fluctuations.

CONTENTS

	Page
CHAPTER I INTRODUCTION	1
1 1 Definition of the Scattering Function	2
1 2 Past Related Work	3
1 3 Organization of the Thesis	5
CHAPTER II ROTATING RANDOM DIPOLE MODEL	6
2 1 Scattering Function Derivation	6
2 2 1 Voltage Reflection coefficient	11
2 2 2 Scattering Function Derivation for the Random Dipole Model	19
2 2 3 Scattering Function Calculation for Gaussian Case	24
2 3 Examples	26
CHAPTER III FLUCTUATING ELLIPSOIDAL SCATTERER MODEL	31
3 1 1 Description of the Model	31
3 1 2 Determination of the Voltage Reflection coefficient	35
3 1 3 Derivation of the Scattering Function	41
3 2 Example	44
CHAPTER IV CONCLUSIONS	50
REFERENCES	52

CHAPTER I

INTRODUCTION

Clutter is a term used to denote the echoes from unwanted radar targets illuminated by the radar beam. Targets are classified as clutter in relation to the application of the radar. Unlike thermal noise, clutter returns are due to radar's own transmission. Therefore increasing the signal power alone without choosing a suitable transmitting waveform does not improve the target detection performance. For designing the transmitting waveform and the optimum radar receiver to operate effectively in the presence of clutter, it is necessary to develop a theoretical model of clutter.

The characteristic that is often chosen to describe the echo signal from extended clutter is the scattering cross section per unit intercepted area σ^0 which is independent of the size of the clutter patch illuminated. This parameter depends on the specific features of the clutter producing targets such as surface roughness, dielectric constant, number of scatterers and the transmitting frequency. But the static cross-section σ^0 is inadequate to characterize the time varying statistical nature of the clutter returns from extended fluctuating targets. A second order description of the clutter return is provided by the scattering function associated with the target along with the envelope of the transmitted waveform.

The scattering function characterisation of clutter which provides a measure of the distribution of clutter power in delay and doppler variables is often considered adequate for designing the transmitted waveform and the optimum receiver

1.1 DEFINITION OF THE SCATTERING FUNCTION

An extended target may be modelled as a random linear time-varying filter and the echo return from such target is then given as the output of the filter whose input is the waveform transmitted by the radar. The complex envelope of the clutter returns can be written as

$$s_r(t) = \int_{-\infty}^{\infty} f(t-\lambda) b((t-\frac{\lambda}{2})-\lambda) d\lambda \quad (1.1)$$

where $\tilde{f}(t-\lambda)$ is the complex envelope of the transmitted signal $b(t-\frac{\lambda}{2}-\lambda)$ is the return from the target located within the range interval $(\lambda, \lambda + d\lambda)$ when an unit impulse is transmitted at $(t-\lambda)$ and λ is the round trip delay time

The correlation function of the received signal assuming $\tilde{s}_r(t)$ has zero mean is given by

$$K_{\tilde{s}_r}(t_\alpha - t_\beta) = E \{ \iint_{-\infty}^{\infty} \tilde{f}(t_\alpha - \lambda) \tilde{f}^*(t_\beta - \lambda) b(t-\frac{\lambda}{2}-\lambda) b^*(t-\frac{\lambda_1}{2}-\lambda_1) d\lambda d\lambda_1 \} \quad (1.2)$$

Assuming that the returns from different interval are statistically independent and the $b(t-\lambda)$ is a stationary random process eqn (1.2) reduces to

$$K_{\tilde{s}_r}(t_\alpha - t_\beta) = \int_{-\infty}^{\infty} \tilde{f}(t_\alpha - \lambda) K_{DR}(t_\alpha - t_\beta - \lambda) \tilde{f}^*(t_\beta - \lambda) d\lambda \quad (1.3)$$

The scattering function $S_{DR}(f, \lambda)$ is defined as

$$S_{DR}(f, \lambda) \triangleq \int_{-\infty}^{\infty} K_{DR}(\tau, \lambda) e^{-j2\pi f \tau} d\tau \quad (14)$$

where $(t_{\alpha} - t_{\beta}) = \tau$

Hence

$$\langle S_r(t_{\alpha} - t_{\beta}) \rangle = \int_{-\infty}^{\infty} f(t_{\alpha} - \lambda) S_{DR}(f, \lambda) \tilde{f}^*(t_{\beta} - \lambda) e^{+j2\pi f \tau} d\tau d\lambda \quad (15)$$

It may be noted that

$$2 \sigma_b^2 = \int_{-\infty}^{\infty} S_{DR}(f, \lambda) df d\lambda$$

represents the ratio of the expected value of the received energy and includes in it the antenna gains, path losses and average radar cross section of the clutter target

1.2 PAST RELATED WORK

In the past some theoretical models which account for the statistical nature of the clutter returns are proposed. Siegert [1], Kerr [8], Larson and Uhlmannbeck [9] presented the first two probability distribution functions of the echo power. Kelly and Lerner [14] developed a theoretical model of a haff cloud from which they derived various statistical properties of the echo. The signal return from the haff cloud was considered as a random process and the scatterers were treated as points with variable cross sections. VanTrees [3] in his model derived an expression for the correlation function of the clutter returns. The returns from the scatterers in the

illuminated volume was assumed to constitute a non homogeneous Poisson process. The strength of the echoes and delays of the scatterers were considered as random variables.

Childers and Reed [4] considered the clutter cloud as collection of point scatterers moving about and reflecting energy independently of one another. The effect of scatterer rotation and multiple scattering were ignored. Amplitude reflection coefficient and the delay of the scatterers were assumed to be random variables. As in the previous model treating the returns from the scatterers in the illuminated volume as a non homogeneous Poisson process the time varying correlation function for a radar back scattered noise process was determined. Subsequently an expression for the power spectrum was obtained assuming the clutter noise process to be stationary.

In the model formulated by Wong, Reed and Kaprielian [1] it was assumed that clutter is an ensemble of random dipoles having linear and angular velocities. An expression for the time varying correlation function of the echo signal was derived in terms of the characteristics of the transmitted waveform polarization and the distribution of the scatterers. An expression for the power spectrum was obtained with suitable assumptions.

J W Wright [2] determined the statistical characteristics of the scattering parameters such as RCS, elevation error, azimuth

error and the target phase of an air raft type target. The target was divided into a finite deterministic number of ellipsoids with varying radar cross sections. The aspect angles of the ellipsoids were treated as random processes to allow fluctuations of the target.

1.3 ORGANIZATION OF THE THESIS

In this thesis two models of clutter are proposed and the scattering function is derived for each. Chapter 2 consists of the first model in which the clutter cloud is treated as random dipoles with linear and angular velocities. The round trip delays are considered as random variables and the echo arrivals are assumed to constitute a non-homogeneous Poisson process. With these assumptions an expression for the scattering function in delay and doppler domain is obtained.

The second model is given in Chapter 3. In this model the clutter cloud is considered as a collection of finite deterministic number of ellipsoidal scatterers. An expression for the voltage reflection coefficient is derived followed by the derivation of the scattering function.

The results of Chapter 2 and Chapter 3 are discussed upon and concluded in Chapter 4.

CHAPTER II

ROTATING RANDOM DIPOLE SCATTERER MODEL

In this chapter an expression for the scattering function is derived where the clutter cloud is assumed to be a collection of random dipole. The dipoles are assumed to be random to represent the random motions of the objects such as leaves, branches etc under the effects of wind forces.

In Section 2.1 scattering function with delay and doppler variables is derived for a collection of random scatterers. Section 2.2 consists of a detailed description of the model, a derivation of the voltage reflection coefficient of a dipole and the derivation of the scattering function of the same model of clutter. It also contains the evaluation of the scattering function where the random variables of interest are assumed to be Gaussian. Some of the experimental results are simulated using the model of section 2.2 and the results of the simulation are reported in Section 2.3.

2.1 SCATTERING FUNCTION DERIVATION

When a signal is transmitted by radar it is scattered by various objects in the illuminated volume which are called scatterers. Therefore the returned signal at any time t is a sum of the echoes from a large number of scatterers. If $\tilde{s}_t(t)$ is the complex envelope of the transmitted signal the

complex envelope of the returned signal $s_r(t)$ is given by

$$\tilde{s}_r(t) = \sum_{j=1}^{N_t} z_j(t - \frac{\tau_j}{2}) s_t(t - \tau_j) \quad (211)$$

where N_t is the number of scatterers which cause an echo to arrive in the interval (T, T) and is assumed to constitute a non homogeneous Poisson process with rate $v(t)$. τ_j is the strength of the echo of the j th scatterer and is assumed to be a stationary random process with zero mean. The processes z_1, z_j for $i \neq j$ are assumed to be statistically independent. Another assumption made here is that the round trip delays τ_j 's are random variables. τ_j 's can be thought of as the unordered delays of the scatterers which give an echo in the interval (T, T) . From the Poisson process assumption given that K echoes have arrived in the interval (T, T) the unordered delays $\tau_1, \tau_2, \dots, \tau_K$ are mutually independent. Since N_t is a non homogeneous Poisson process they are assumed to have a common density function of the form

$$f_{\tau_j}(\tau_j / N_t = K) = \frac{\lambda^K}{\int_T^T v(t) dt} \quad (212)$$

Since the scattering function should be evaluated in delay and doppler domain it is necessary to calculate the auto correlation of the returned signal where the returned signal is a sum of the echoes from all those scatterers which

contribute to the same delay i.e. the random variables τ_j $j = 1 \dots K$ should all take a single value say λ in the interval (T, T)

The probability that the random variable τ_j takes a value between $(\lambda, \lambda + d\lambda)$ where $d\lambda \rightarrow 0$ is

$$P_{\tau_j} \{ \lambda \leq \tau_j < \lambda + d\lambda \} = \frac{v(\lambda) d\lambda}{\int_0^T v(t) dt} \quad (2.1.3)$$

Autocorrelation of the received signal is

$$R_{S_r} (t_\alpha - t_\beta) = E \{ S_r(t_\alpha) \tilde{S}_r^*(t_\beta) \} \quad (2.1.4)$$

$$= \sum_{k=0}^{\infty} E \{ \tilde{S}_r(t_\alpha) S_r^*(t_\beta) / N_t = k \} \Pr [N_t = k] \quad (2.1.5)$$

$$\lambda < \tau_j < \lambda + d\lambda \quad \lambda < \tau_j < \lambda + d\lambda$$

$$= E \{ \sum_{j=1}^K \sum_{l=1}^K \tilde{Z}_j(t_\alpha - \frac{\tau_j}{2}) \tilde{Z}_l(t_\beta - \frac{\tau_l}{2}) \tilde{S}_r(t_\alpha - \tau_j) \tilde{S}_r^*(t_\beta - \tau_l) / N_t - k \} \quad (2.1.6)$$

$$\lambda < \tau_j < \lambda + d\lambda$$

Since \tilde{Z}_j 's are independent with zero mean $\forall j$
eqn (2.1.6) reduces to

$$E \{ \tilde{S}_r(t_\alpha) \tilde{S}_r^*(t_\beta) / N_t = K \} \mid$$

$$\lambda < \tau_j < \lambda + d\lambda$$

$$= \sum_{j=1}^K E \{ [\Delta_j(t_\alpha - \frac{\tau}{2}) \tilde{Z}_j^*(t_\beta - \frac{\tau}{2}) S_t(t_\alpha - \tau_j) S_t^*(t_\beta - \tau_j)] / N_t = K \} \mid \lambda < \tau_j < \lambda + d\lambda \quad (2.1.7)$$

$$= \sum_{j=1}^K E \{ Z_j(t_\alpha - \frac{\lambda}{2}) Z_j^*(t_\beta - \frac{\lambda}{2}) \} \frac{\nu(\lambda) d\lambda}{\int_T \nu(t) dt} S_t(t_\alpha - \lambda) S_t^*(t_\beta - \lambda) \\ = K E \{ Z_j(t_\alpha - \frac{\lambda}{2}) Z_j^*(t_\beta - \frac{\lambda}{2}) \} \frac{\nu(\lambda) d\lambda}{\int_T \nu(t) dt} S_t(t_\alpha - \lambda) \tilde{S}_t^*(t_\beta - \lambda) \quad (2.1.8)$$

where Z_j 's are assumed to be identically distributed

Assuming the process Z_j to be stationary

$$E \{ \tilde{S}_r(t_\alpha) \tilde{S}_r^*(t_\beta) / N_t = K \} d\lambda = K R_Z(\tau) \frac{\nu(\lambda) d\lambda}{\int_T \nu(t) dt} \tilde{S}_t(t_\alpha - \lambda) S_t^*(t_\beta - \lambda) \quad (2.1.9)$$

Since

$$\lim_{d\lambda \rightarrow 0} E \{ \tilde{S}_r(t_\alpha) \tilde{S}_r^*(t_\beta) / N_t = K \} = E \{ \tilde{S}_r(t_\alpha) \tilde{S}_r^*(t_\beta) / N_t = K \} d\lambda \quad (2.1.10)$$

$$\lambda < \tau_j < \lambda + d\lambda$$

$$R_r(t_\alpha - t_\beta - \lambda) = R_A(\tau) \tilde{S}_t(t_\alpha - \lambda) \tilde{S}_t^*(t_\beta - \lambda) \frac{\nu(\lambda)}{\int_T \nu(t) dt} \sum_{K=0}^{\infty} I \Pr[N_t = K] \\ = R_Z(\tau) \tilde{S}_t(t_\alpha - \lambda) S_t^*(t_\beta - \lambda) \gamma(\lambda) \quad (2.1.11)$$

$$\text{because } \sum_{K=0}^{\infty} K \Pr[V_t=K] = \int_0^T v(t) dt \quad (2.1.12)$$

From (2.1.11) the auto correlation of the process $\tilde{v}(t)$ is $R(\tau)$

$$\text{Let } R_x(\tau) \stackrel{\Delta}{=} R(\tau) v(\lambda) \quad (2.1.13)$$

Then the normalized correlation function is

$$\xi(\tau) = \frac{R_x(\tau)}{R_x(0)} \quad (2.1.14)$$

Therefore the scattering function is given by

$$(f(\lambda) = \int_{-\infty}^{\infty} g(\tau \lambda) e^{-j2\pi f\tau} d\tau \quad (2.1.15)$$

Random dipole model for radar clutter was initially suggested in [1] by J L Long I S Reed et al in which an expression for the fluctuation frequency spectrum was derived. For the same model in the present work a different and an easier approach is taken to calculate the voltage reflection coefficient an expression for the scattering function in delay and doppler domain is derived.

In this model it is assumed that the illuminated volume consists of a collection of random dipole scatterers reflecting energy independently the cloud of scatterers have an overall drift velocity and the scatterers have differential velocities. In addition to these it is also assumed that the scatterers have rotational motion about an axis perpendicular to the dipole axis. For land clutter rotational motion or equivalent movement of branches leaves etc under the

effects of windforces is a major contributing factor to the fluctuation of echo intensity which can not be neglected. The rate of echo arrival depend on the local density of the cloud of scatterers.

2.2.1 VOLTAGE REFLECTION COEFFICIENT

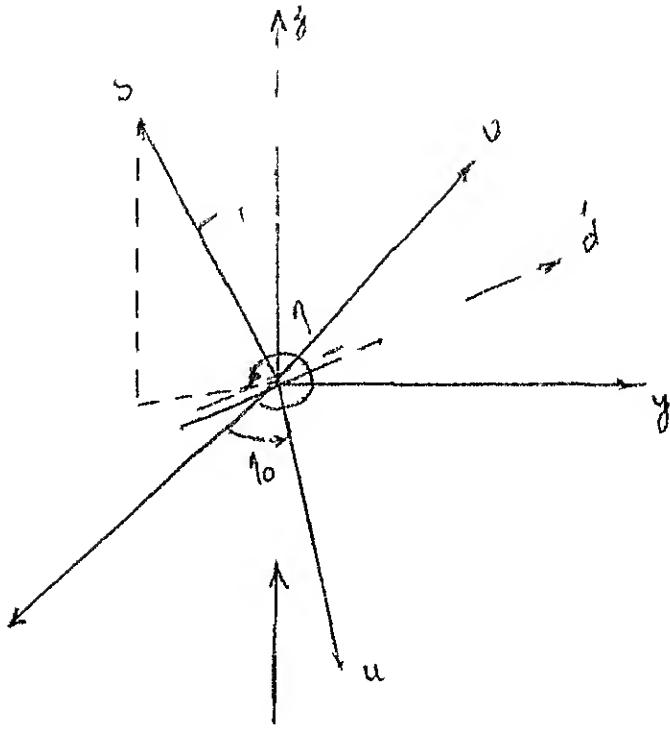
To calculate the voltage reflection coefficient for a dipole whose axis is oriented parallel to the unit vector d in the target fixed coordinate system as shown in Fig (1) it is assumed that the dipole is rotating about an axis S which is perpendicular to the axis of the dipole. The axis S is defined by the angle ξ and η as shown. Let $(u v s)$ be a set of orthogonal axes such that the plane $u v$ is the same as the plane of rotation of the dipole. Therefore the transformation between the coordinate systems where η_0 and ξ are right handel angles about z and the new x axis respectively is given by

$$\begin{bmatrix} a_u \\ a_v \\ a_s \end{bmatrix} = \begin{bmatrix} \sin \eta & \cos \eta & 0 \\ \cos \xi & \cos \eta & \cos \xi \sin \eta \\ \sin \xi \cos \eta + \sin \xi \sin \eta & \cos \xi \end{bmatrix} \begin{bmatrix} \hat{i}_t \\ \hat{j}_t \\ \hat{k}_t \end{bmatrix} \quad (2.2.1)$$

$$\text{where } \eta_0 = \eta - 270 \quad (2.2.2)$$

a_u a_v a_s are unit vector along u v and s respectively and

i_t j_t k_t are unit vectors along x y and z respectively



Direction of the incident wave

Fig (1)

Let ω_r be the angular frequency of the dipole and α be the initial angle of the dipole with respect to the u axis. Then the instantaneous position of the unit vector \hat{d} is given by

$$\hat{d} = a_u \cos \psi + a, \text{ in } \psi \quad (223)$$

where

$$\psi = \omega_r t + \alpha \quad (224)$$

From (221) and (223)

$$\begin{aligned} \hat{d} = & -i_t (\sin \eta \cos \psi + \cos \xi \cos \eta \sin \psi) \\ & + j_t (\cos \eta \cos \psi - \cos \xi \sin \eta \sin \psi) \quad \hat{i}_t (\sin \xi \sin \psi) \end{aligned} \quad (225)$$

When an RF signal is transmitted by the radar it produces an electric field at the dipole which induces a current in the dipole. The dipole reradiates energy due to the current induced in it thus producing an electric field at the radar receiver.

Let \hat{E}_{inc} be the field incident on the dipole which is given by

$$\hat{E}_{\text{inc}} = E_{\text{OT}} e_T \quad (226)$$

where E_{OT} is the magnitude of the electric field and e_T is the transmission polarization vector.

If ϕ_0 is the polarization angle with respect to the x axis

$$e_T = \hat{i}_t \cos \phi_0 + j_t \sin \phi_0 \quad (227)$$

The current density J induced in the dipole is given by

$$J = j \omega_c \epsilon_0 (r - 1) E_{OT} (e_T \hat{a}) \quad (2.2.8)$$

where ω_c is the frequency of transmission ϵ_0 is the permittivity of free space and ϵ_r is the relative permittivity of the medium

The electric field produced by the dipole at a point which is at a distance r from the dipole is

$$\vec{E}_{ref} = v^2 \vec{\Pi} + \text{grad} (\text{div} \vec{\Pi}) \quad (2.2.9)$$

where

$$v^2 = (\epsilon \mu \omega_c^2 + j \sigma \mu \omega_c) \quad (2.2.10)$$

μ is the permeability of the medium and

σ is the conductivity of the medium

The Hertz vector $\vec{\Pi}$ is given by [10]

$$\vec{\Pi} = \frac{J}{4\pi(\sigma + j\omega_c \epsilon)} \frac{e^{-vr}}{r} \hat{d} \quad (2.2.11)$$

The operations grad and div are

$$\text{grad } A \triangleq \nabla A \triangleq \frac{\partial A}{\partial x} \hat{i} + \frac{\partial A}{\partial y} \hat{j} + \frac{\partial A}{\partial z} \hat{k} \quad (2.2.12)$$

$$\text{div } \vec{A} \triangleq \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad (2.2.12)$$

From (2.2.8) and (2.2.11)

$$\vec{\Pi} = \frac{j \omega_c \epsilon_0 (\epsilon_r - 1)}{4\pi(\sigma + j\omega_c \epsilon)} E_{OT} \frac{e^{-vr}}{r} (e_T \hat{a}) \hat{d} \quad (2.2.13)$$

Rewriting (2 2 5)

$$\begin{aligned} \mathbf{d} = & -\mathbf{i}_t (\sin \eta \cos \psi + \cos \xi \cos \eta \sin \psi) \\ & + \mathbf{j}_t (\cos \eta \cos \psi - \cos \xi \sin \eta \sin \psi) + \mathbf{k}_t (\sin \xi \sin \psi) \end{aligned}$$

Let

$$A \triangleq \sin \eta \cos \psi \quad \cos \xi \cos \eta \sin \psi$$

$$B \triangleq \cos \eta \cos \psi \quad \cos \xi \sin \eta \sin \psi \quad (2 2 14)$$

$$C \triangleq \sin \xi \sin \psi$$

Then the unit vector \mathbf{d} can be written as

$$\mathbf{d} = \mathbf{i}_t A + \mathbf{j}_t B + \mathbf{k}_t C \quad (2 2 15)$$

From (2 2 7) and (2 2 15)

$$d \mathbf{e}_T = A \cos \phi_0 + B \sin \phi_0 \quad (2 2 16)$$

Let

$$K = \frac{j \omega_c \epsilon_0 (\epsilon_r - 1) E_{OT}}{4 \pi (\sigma + j \omega_c \epsilon)} \quad (2 2 17)$$

and

$$K_1 = K (A \cos \phi_0 + B \sin \phi_0) \quad (2 2 18)$$

Substituting (2 2 17) and (2 2 18) in (2 2 13)

$$\vec{H} = K_1 \frac{e^{j\omega_c t}}{r} (\mathbf{i}_t A + \mathbf{j}_t B + \mathbf{k}_t C) \quad (2 2 19)$$

Substituting $r = \sqrt{x^2 + y^2 + z^2}$

$$\begin{aligned} \frac{\partial \vec{\Pi}}{\partial x} &= K_1 A \frac{\partial}{\partial x} \left| \frac{e^{v/\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2} \right| \\ &= K_1 A \left| \frac{e^{v/\sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)^2} \right. x \left(v - \frac{1}{\sqrt{x^2+y^2+z^2}} \right) \left. \right| \end{aligned}$$

Similarly

$$\begin{aligned} \frac{\partial \vec{\Pi}}{\partial y} &= K_1 B \left| \frac{e^{v/\sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)^2} \right. y \left(v - \frac{1}{\sqrt{x^2+y^2+z^2}} \right) \left. \right| \quad (2.2.20) \\ \frac{\partial \vec{\Pi}}{\partial z} &= K_1 C \left| \frac{e^{v/\sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)^2} \right. z \left(v - \frac{1}{\sqrt{x^2+y^2+z^2}} \right) \left. \right| \end{aligned}$$

Therefore

$$\operatorname{div} \vec{\Pi} = K_1 \left| \frac{e^{v/\sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)^2} \right. \left(v - \frac{1}{\sqrt{x^2+y^2+z^2}} \right) (xA + yB + zC) \quad (2.2.21)$$

$$\operatorname{grad}(\operatorname{div} \vec{\Pi}) = \frac{\partial}{\partial x}(\operatorname{div} \vec{\Pi}) \hat{i}_t + \frac{\partial}{\partial y}(\operatorname{div} \vec{\Pi}) \hat{j}_t + \frac{\partial}{\partial z}(\operatorname{div} \vec{\Pi}) \hat{k}_t \quad (2.2.22)$$

$$\begin{aligned} \frac{\partial}{\partial x}(\operatorname{div} \vec{\Pi}) &= K_1 A \left\{ \frac{e^{v/r}}{r^2} \left(v + \frac{1}{r} \right) \right. \\ &\quad \left. - x^2 \left[\frac{e^{v/r}}{r^2} \frac{x}{r^3} + x \left(v + \frac{1}{r} \right) \left(\frac{e^{v/r}}{r^3} - \frac{2e^{v/r}}{r^4} \right) \right] \right\} \quad (2.2.23) \end{aligned}$$

Similarly $\frac{\partial}{\partial y}(\operatorname{div} \vec{\Pi})$ $\frac{\partial}{\partial z}(\operatorname{div} \vec{\Pi})$ can be determined

From (2.2.23) it can be seen that $\operatorname{grad}(\operatorname{div} \vec{\Pi})$ contains only

$\frac{1}{r^2}$ $\frac{1}{r^3}$ $\frac{1}{r^4}$ and $\frac{1}{r^5}$ terms Hence grad (div \vec{n}) $\neq 0$ for large values of

$$\text{Therefore } \vec{E}_{\text{ref}} = -v^2 \vec{n} = v^2 K \frac{e^{-v}}{r} (A \cos \phi_0 + B \sin \phi_0) \\ (1_t A + \hat{j}_t B + k_t C) \quad (2.2.24)$$

The electric field received by the radar due to the scattering by the dipole is given by

$$E_{\text{rec}}^s = E_x^s \cos \phi_r + L_j^s \sin \phi_r \quad (2.2.25)$$

where ϕ_r is the receiver polarization with respect to the y axis

$$E_x^s = v^2 I A \frac{e^{-v}}{r} (A \cos \phi_0 + B \sin \phi_0) \quad (2.2.26)$$

and

$$E_y^s = v^2 K B \frac{e^{-v}}{r} (A \cos \phi_0 + B \sin \phi_0)$$

From the above

$$L_j^s_{\text{rec}} = \frac{(\epsilon \mu \omega_c^2 + j \sigma \mu \omega) j \omega_c \epsilon_0 (\epsilon_r - 1) E_{\text{OT}}}{4\pi(\sigma + j \omega_c \epsilon_r)} \frac{e^{-v}}{r} \\ (A \cos \phi_0 + B \sin \phi_0) (A \cos \phi_r - B \sin \phi_r) \quad (2.2.27)$$

Assuming the medium to be lossless and where the radar is monostatic

$$\sigma = 0 \quad \text{and} \quad \phi_r = \phi_0 \quad (2.2.28)$$

$$E_{\text{rec}}^s = K_2 E_{\text{OT}} \frac{e^{-j\omega\mu\epsilon r}}{r} (A \cos \phi_o + B \sin \phi_o)^2 \quad (2.2.29)$$

where

$$K_2 = \frac{\mu \omega c \epsilon_o (\epsilon_r - 1)}{4} \quad (2.2.30)$$

Radar cross section for the j th scatterer is defined as

$$\sigma_j \triangleq \lim_{\substack{r_j \rightarrow \infty \\ r_j \rightarrow \infty}} \frac{r_j^2}{4\pi} \left| \frac{E_{\text{ref}}^s}{E_{\text{inc}}^s} \right| \quad (2.2.31)$$

From (2.2.26) and (2.2.19)

$$\left| E_{\text{inc}}^s \right| = E_{\text{OT}}$$

$$\left| E_{\text{rec}}^s \right| = \frac{K_2 E_{\text{OT}}}{r_j} (A_j \cos \phi_o + B_j \sin \phi_o)^2$$

Hence

$$\sigma_j = 4\pi K_2^2 \left[(A_j \cos \phi_o + B_j \sin \phi_o)^2 \right]^2 \quad (2.2.31)$$

The voltage reflection coefficient is related to the RCS as given below

$$\left| \bar{V}_j^2 \right| = \frac{G^2 \lambda_c^2}{(4\pi)^2 r_j^4} \left| \bar{\sigma}_j \right| \quad (2.2.32)$$

where V_j - is voltage reflection coefficient
and λ_c is the radar wavelength

ubstituting (2 2 31) in (2 2 32)

$$|v_J| = \frac{K_3}{r_J^2} (A_J \cos \phi_0 + B_J \sin \phi_0)^2 \quad (2 2 33)$$

where

$$K_3 = \frac{G \lambda_c \mu \omega^2 \epsilon_0 (\epsilon_r - 1)}{(4\pi)^2}$$

Therefore

$$\tilde{v}_J = \frac{r_J}{r_J^2} (A_J \cos \phi_0 + B_J \sin \phi_0)^2 e^{j\beta}$$

where β is the random phase of the reflection coefficient incurred in the reflection process

It can be shown that

$$(A_J \cos \phi_0 + B_J \sin \phi_0)^2 = \tilde{C}(\xi_J, \eta_J) + \tilde{U}(\xi_J, \eta_J) e^{j2\psi_J} + \tilde{L}(\xi_J, \eta_J) e^{-j2\psi_J}$$

where

$$\psi_J = \omega_{r_J} t + \alpha_J$$

$$\tilde{C} = \frac{1}{4} (S^2 - R^2 \cos^2 \xi_J)$$

$$\tilde{U} = (S + R \cos \xi_J)^2$$

$$\tilde{L} = \frac{1}{4} (R^2 \cos^2 \xi_J)$$

$$R = [\cos \phi_0 \cos \eta_J + \sin \phi_0 \sin \eta_J]$$

$$S = K_J [\cos \phi_0 \sin \eta_J - \sin \phi_0 \cos \eta_J]$$

$$\tilde{v}_J = \frac{K_3}{r_J^2} [\tilde{C}(\xi_J, \eta_J) + \tilde{U}(\xi_J, \eta_J) e^{j2\psi_J} + \tilde{L}(\xi_J, \eta_J) e^{-j2\psi_J}] e^{j\beta}$$

2 2 2 SCATTERING FUNCTION DERIVATION FOR THE RANDOM DIPOLE MODEL

Let $s_t(t)$ be the complex envelope of the transmitted signal. Then the complex envelope of the received signal from the j th scatterer can be written as

$$r_j(t) = v_j(t - \frac{\tau_j(t)}{2}) \tilde{s}_t(t - \tau_j(t)) \exp[j\omega_c \tau_j(t) + j\beta] \quad (2.2.34)$$

where $v_j(t)$ is the reflection coefficient of the j th scatterer, $\tau_j(t)$ is the round trip delay of the j th scatterer, ω_c is the carrier frequency and β is the random phase incurred in the reflection process.

In the beginning of this section it was assumed that the scatterers have an overall drift velocity and also differential velocities. Let v_j be the total velocity of the j th scatterer in the radial direction. The velocities of the scatterers in all the other directions can be neglected since their doppler contribution to the echo signal is very small. Therefore the range of the scatterer at time t is given by

$$r_j(t) = r_{j0} + v_j t \quad (2.2.35)$$

Round trip delay time $\lambda_j(t)$ implies that a signal received at time t was reflected from the scatterer at time $(t - \lambda_j(t)/2)$. At that instant the range of the scatterer was

$$r_j(t - \frac{\lambda_j(t)}{2}) = r_{j0} - v_j(t - \frac{\lambda_j(t)}{2})$$

$$\lambda_j(t) \triangleq \frac{2 r_j(t - \frac{\lambda_j(t)}{2})}{c}$$

where c is the velocity of light

from the above two eqns

$$\lambda_j(t) = \frac{2 r_0 /}{1 + v_j / c} - \frac{(2 v_j / c)t}{1 + v_j / c}$$

assuming that $\frac{v_j}{c} \ll 1$

$$\lambda_j(t) = \frac{2 r_0}{c} - \frac{2 v_j}{c} t - \lambda_j - \frac{2 v_j}{c} t \quad (2.2.36)$$

assuming that the cloud of scatterers is very far from the radar

$$\frac{r^2}{k} = r_1^2 - r_0^2$$

Therefore

$$\begin{aligned} \tilde{s}_{r_j}(t) &= \frac{K_3}{r_0^2} \left[\tilde{r}(\xi_j, n_j) + \tilde{U}(\xi_j, n_j) e^{j2\omega_r(t - \frac{\lambda_j(t)}{2})} + \alpha_j \right] \\ &+ L(\xi_j, n_j) e^{j2 \left[\omega_r(t - \frac{\lambda_j(t)}{2}) + \alpha_j \right]} \\ s_t(t - \lambda_j) &\propto \exp[j\omega_c \tau_j(t) + j\beta] \quad (2.2.37) \end{aligned}$$

Assuming β to be uniformly distributed in $(0, 2\pi)$ it can be seen that \tilde{s}_{r_j} has zero mean

The total received signal at time t can be written as

$$s_r(t) = \sum_{j=1}^{N_t} \tilde{s}_{r_j}(t) \quad (2.2.38)$$

Auming that N_t is a non homogeneous Poisson process with rate $v(t)$ the process is stationary and λ_j 's are the unordered delays the autocorrelation of the received signal which follow from (2.1.11) can be written as

$$R_r(t_\alpha - t_\beta - \lambda) = R(1) v(\lambda) \tilde{S}_t(t_\alpha - \lambda) S_t^*(t_\beta - \lambda) \quad (2.2.39)$$

where

$$\begin{aligned} R_Z(t - \tau - t - \lambda) &= E \left\{ \frac{K^2}{4} \left[C(\xi, \eta) + U(\xi, \eta) e^{j(2\omega_r(t - \tau - \frac{(t - \lambda)}{2}) + 2\alpha)} \right. \right. \\ &\quad + j(2\omega_r(t - \tau - \frac{\lambda(t - \tau)}{2} + 2\alpha)) \\ &\quad + L(\xi, \eta) e^{j(2\omega_r(t - \frac{\lambda(t)}{2} + 2\alpha))} \\ &\quad \left. \left. + C(\xi, \eta) + U(\xi, \eta) e^{-j(2\omega_r(t - \frac{\lambda(t)}{2} + 2\alpha))} \right] \right\} \\ &= p [j\omega_c \lambda_j(t - \tau) - j\omega_c \lambda_j(t)] \end{aligned}$$

and Z_j 's are assumed to be uncorrelated and identically distributed

$$\begin{aligned} R_Z^*(t - \tau - t - \lambda) &= E \left\{ \frac{K^2}{4} [|C^2(\xi, \eta)| + |U^2(\xi, \eta)| e^{+j2\omega_r \tau} \right. \\ &\quad \left. + |L^2(\xi, \eta)| e^{j2\omega_r \tau}] \exp(j\omega_d \tau) \right\} \quad (2.2.40) \end{aligned}$$

$$\text{where } \omega_d = \frac{2\omega_c v_j}{c}$$

$$\text{and } \frac{\omega_r v_j}{c} \ll 1$$

Therefore

$$\begin{aligned}
 R_L(\tau) &= \frac{K_3^2}{r_0^4} \iiint_{-\infty}^{\infty} p(\omega_d) p(\omega_r) p(\xi) p(\eta) d\omega_d d\omega_r d\xi d\eta \\
 &[|U^2(\xi \eta)| + |U^2(\xi \eta)| e^{+j2\omega_r \tau} + \\
 &+ |L^2(\xi \eta)| e^{j2\omega_r \tau}] \exp(j\omega_d \tau) \quad (2.2.41)
 \end{aligned}$$

where ω_d , ω_r , ξ and η are assumed to be independent

The scatterers can move towards or away from the radar along the line of sight with equal probability and with their velocity distributions centred about $\pm \bar{\omega}_d$. Let $P(E)$ be the probability of the event ω_d being positive and $P(\bar{E})$ is the probability of the event ω_d being negative

Since these probabilities are equal $P(E) = P(\bar{E}) = \frac{1}{2}$

$$\omega_d = x + \bar{\omega}_d$$

$$P(\omega_d | \omega_d < \omega_d) = P(\omega_d | E) [\omega_d / (E)] P(E)$$

$$+ P(\omega_d | \bar{E}) [\omega_d / (\bar{E})] P(\bar{E})$$

$$E\omega_d(\omega_d) = P_x[\omega_d - \bar{\omega}_d] + \frac{1}{2} P_x[\omega_d + \bar{\omega}_d]$$

$$p\omega_d(\omega_d) = \frac{1}{2} p_x(\omega_d - \bar{\omega}_d) + \frac{1}{2} p_x(\omega_d + \bar{\omega}_d)$$

$$\begin{aligned}
 & \int_{-\infty}^{\infty} p_{\text{d}}(\omega_{\text{d}}) e^{+j\omega_{\text{d}} \tau} - \frac{1}{2} \int_{-\infty}^{\infty} p_x(\omega_{\text{d}} - \omega_{\text{d}}) e^{+j\omega_{\text{d}} \tau} d\omega_{\text{d}} \\
 & + \int_{-\infty}^{\infty} p_x(\omega_{\text{d}} + \bar{\omega}_{\text{d}}) e^{j\omega_{\text{d}} \tau} d\omega_{\text{d}} \\
 & \approx \cos(\bar{\omega}_{\text{d}} \tau) \quad (2.2.42)
 \end{aligned}$$

Assuming that each orientation of the rotation axis is equally probable

$$\langle x^2 \rangle = \int_0^{2\pi} \int_0^{\pi} x^2 \sin \xi d\xi d\eta \quad (2.2.43)$$

where x^2 represents $|\sigma^2(\xi, \eta)|$ or $|U^2(\xi, \eta)|$ and $|L^2(\xi, \eta)|$

From the above (2.2.41) reduces to

$$R_x(\tau) = \frac{K_3^2}{r_0^4} [\langle \sigma^2 \rangle + \langle U^2 \rangle \downarrow \omega_x(2\tau) + \langle L^2 \rangle \downarrow \omega_x(2\tau)] \phi_{\omega_d}(\tau) \cos(\omega_d \tau) \quad (2.2.45)$$

where

$$\phi_{\omega_x}(\tau) = \int_{-\infty}^{\infty} p(\omega_x) e^{j\omega_x \tau} d\omega_x \quad (2.2.46)$$

From (2.1.13) and (2.2.45)

$$\begin{aligned}
 R_x(\tau, \lambda) &= \frac{K_3^2}{r_0^4} [\langle \sigma^2 \rangle + \langle U^2 \rangle \phi_{\omega_x}(2\tau) \\
 &+ \langle L^2 \rangle \phi_{\omega_x}(2\tau)] \phi_{\omega_d}(\tau) \cos(\omega_d \tau) v(\lambda) \quad (2.2.47)
 \end{aligned}$$

Hence the normalised correlation function is

$$g(\tau \lambda) = \frac{1}{\langle b^2 \rangle} [\langle c^2 \rangle + \langle u^2 \rangle \phi_{\omega_r}(2\tau) + \langle L^2 \rangle \phi_{\omega_r}(2\tau)] \phi_{\omega_d}(\tau) \cos(\bar{\omega}_d \tau) \quad (2.2.48)$$

$$\text{where } \langle b^2 \rangle = \langle c^2 \rangle + \langle u^2 \rangle + \langle L^2 \rangle \quad (2.2.49)$$

From (2.2.48) the scattering function can be written as

$$S(f \lambda) = \int_{-\infty}^{\infty} g(\tau \lambda) e^{-j2\pi f \tau} d\tau \quad (2.2.50)$$

2.2.3 SCATTERING FUNCTION CALCULATION FOR GAUSSIAN CASE

In this sub section scattering function is calculated assuming that the doppler frequency ω_d and the angular frequency ω_r have Gaussian probability distributions

Let

$$p_{\omega_d}(\omega_d) = \frac{1}{\sqrt{2\pi\sigma_d^2}} \exp [(\omega_d - \bar{\omega}_d)^2 / 2\sigma_d^2] \quad (2.2.51)$$

and

$$p_{\omega_r}(\omega_r) = \frac{1}{\sqrt{2\pi\sigma_r^2}} \exp [(\omega_r - \bar{\omega}_r)^2 / 2\sigma_r^2] \quad (2.2.52)$$

$$\psi_{\omega_d}(\tau) = \exp(j\bar{\omega}_d \tau) \exp(-\sigma_d^2 \tau^2) \quad (2.2.53)$$

$$\psi_{\omega_r}(\tau) = \exp(j\bar{\omega}_r \tau) \exp(-\sigma_r^2 \tau^2) \quad (2.2.54)$$

The values of $\langle c^2 \rangle$, $\langle u^2 \rangle$ and $\langle L^2 \rangle$ are calculated in [1] for the following special cases 1) linear transmit linear receive 2) circular transmit circular receive

3) orthogonal linear transmit and receive 4) orthogonal circular transmit and receive They are given as follows

case	$\langle c^2 \rangle$	$\langle U^2 \rangle$	$\langle L^2 \rangle$	$\langle b^2 \rangle$
1	$\frac{2}{15}$	$\frac{1}{30}$	$\frac{1}{20}$	$\frac{1}{5}$
2	$\frac{1}{30}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{2}{15}$
3	$\frac{1}{60}$	$\frac{1}{40}$	$\frac{1}{40}$	$\frac{1}{15}$
4	$\frac{7}{60}$	$\frac{1}{120}$	$\frac{1}{120}$	$\frac{2}{15}$

From the above it can be seen that $\langle U^2 \rangle = \langle L^2 \rangle$

Substituting (2.2.53) and (2.2.54) in (2.2.48)

$$g(\tau, \lambda) = \frac{1}{\langle b \rangle} [\langle c^2 \rangle + 2 \langle s^2 \rangle \exp(-2 \sigma_r^2 \tau^2) \cos(2 \bar{\omega}_r \tau)] \cos(\bar{\omega}_d \tau) \exp(-\frac{1}{2} \sigma_d^2 \tau^2) \quad (2.2.55)$$

$$\text{where } \langle L^2 \rangle = \langle U^2 \rangle - \langle c^2 \rangle$$

The phase factor $e^{j\omega_d \tau}$ is suppressed because it along with $e^{j\omega_c \tau}$ correspond to the heterodyning of the received signal with an oscillator of frequency $(\omega_c + \bar{\omega}_d)$

$$S(f, \lambda) = \int_{-\infty}^{\infty} \frac{1}{\langle b \rangle} [\langle c^2 \rangle + 2 \langle s^2 \rangle \exp(-2 \sigma_r^2 \tau^2) \cos(2 \bar{\omega}_r \tau)] \cos(\bar{\omega}_d \tau) \exp(-\frac{1}{2} \sigma_d^2 \tau^2) e^{-j2\pi f \tau} d\tau \quad (2.2.56)$$

SECTION 2 3 LXA TILS

In this section some of the results simulated using the model discussed in previous section are compared with those obtained experimentally. In the past experiments were conducted to obtain the frequency spectrum of power fluctuations where the power return $P(t)$ was treated as a random process because of the random fluctuations of the scatterers. The correlation function of the power return was found to be [9]

$$\rho(\tau) = \frac{\overline{P(t)P(t+\tau)}}{\overline{P^2(t)}} = g^2(\tau) \quad (231)$$

where $g(\tau)$ is the correlation function of the echo voltage

$\rho(\tau)$ was determined by the interpolation of the discrete observation and the power spectrum which is the Fourier transform of $\rho(\tau)$ was plotted. For comparison it is necessary to find out $\rho(\tau)$ from the results obtained in Section 2 from (2255) $g(\tau)$ is given by

$$g(\tau) = \frac{1}{b^2} [\langle e^2 \rangle + 2 \langle s^2 \rangle \exp(-2 \sigma_r^2 \tau^2) \cos(2 \bar{\omega}_r \tau)] \cos(\bar{\omega}_d \tau) \exp(-\frac{1}{2} \sigma_d^2 \tau^2) \quad (232)$$

$$\rho(\tau) = \left[\frac{\langle e^2 \rangle}{b^2} + \frac{2 \langle s^2 \rangle}{b^2} \exp(-2 \sigma_r^2 \tau^2) \cos(2 \bar{\omega}_r \tau) \right]^2 \cos^2(\bar{\omega}_d \tau) \exp(-\sigma_d^2 \tau^2) \quad (233)$$

$$S_1(f) = \int_{-\infty}^{\infty} \rho(\tau) e^{j2\pi f\tau} d\tau \quad (234)$$

$S_1(f)$ in eqn (234) is simulated and is compared with a few power spectra obtained experimentally

In 1949 E J Barlow [6] reported spectra for wooded hill echo rain clouds and haff measured at a frequency 1 GHz. According to Barlow clutter power spectra can be presented by Gaussian shaped curves of the form

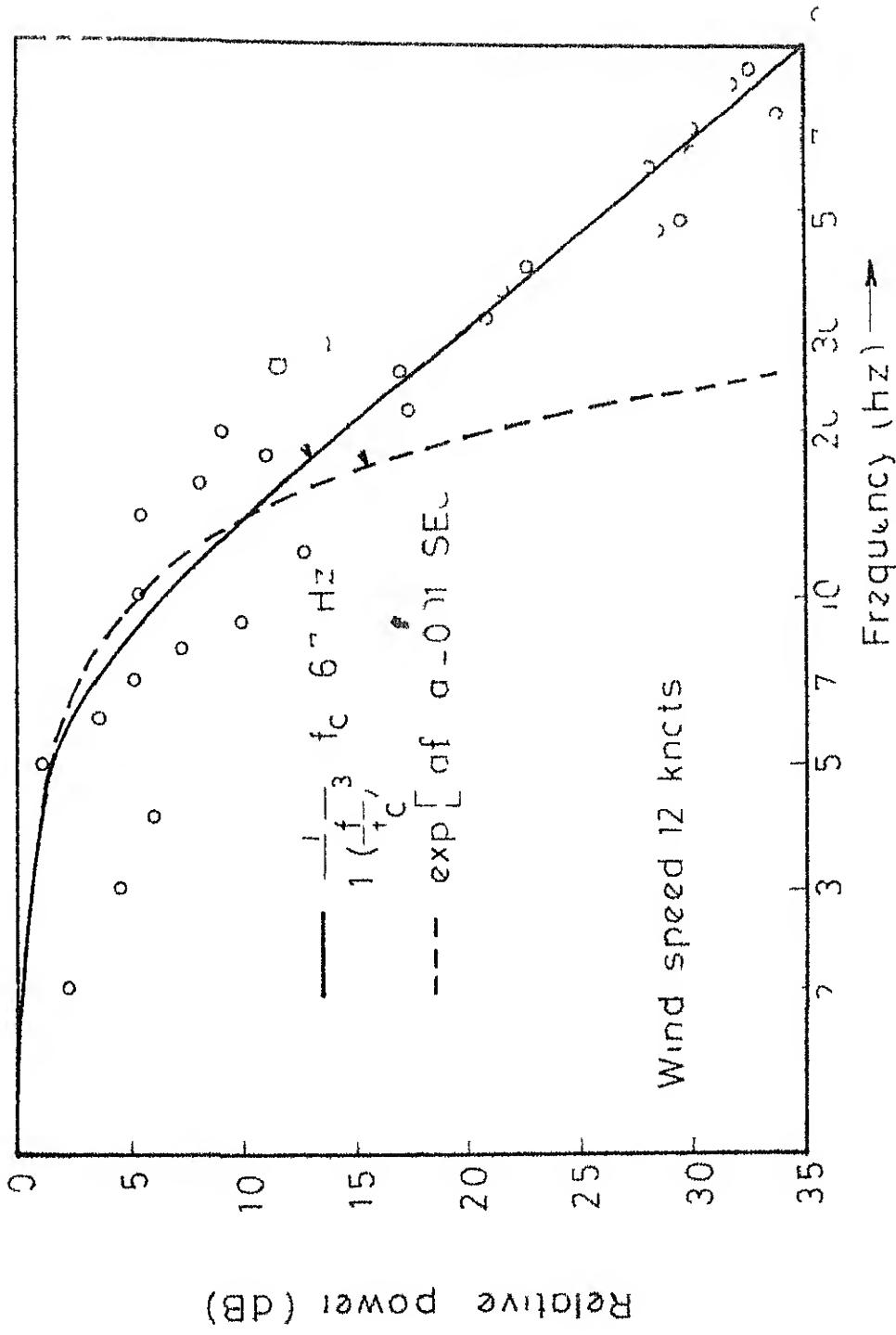
$$\tau(f) = \exp \left[-a \left(\frac{f}{f_0} \right)^2 \right] \quad (235)$$

where f_0 is the radar transmitter frequency and a is a parameter dependent on the radar target type. In practice it was observed by Hbein, Grav line et al that the power spectrum does not take Gaussian shape. According to them a simple expression that gives good agreement with their measured spectra for deciduous foliage is

$$P(f) = \frac{1}{1 + \left(\frac{f}{f_c} \right)^3} \quad (236)$$

where $f_c = 1.33 \times 10^{13.6} v$ and v is the wind speed in knots

For their experiment on tree echo in 12-knot wind F. Hbein, Graveline et al used an x band coherent pulse radar with horizontal polarization and the experimental results are shown by curve a in Fig (2). The Gaussian shaped curve b illustrates the results calculated using eqn (235). From



Source Fishbein Graveline and Rittner 1977

Fig 2 Power spectrum obtained with an X band radar for 12 knot wind speed measurement

Fig (2) it can be clearly seen that the Gaussian curve falls off faster at higher frequencies than does the experimental curve. In the result are simulated for linear polarization and the simulation is done by trial and error.

EXAMPLE 1

Here an attempt is made to simulate results predicted by L J Barlow. In Fig (3) curve a b are the results reported by E T Barlow, and the curves a b illustrate the simulated results. The parameters for the curves a and b are as follows

For the curve - a

Mean value of the doppler shift - $\bar{\omega}_d = 5.3$ rad/sec

Variance of the doppler shift = $\sigma_d = 0.15$ rad/sec

Mean value of the angular frequency $\bar{\omega}_r = 2.4$ rad/sec

Variance of the angular frequency $\sigma_r = 1.2$ rad/sec

For the curve b

$\bar{\omega}_d = 13.3$ rad/sec

$\sigma_d = 10.9$ rad/sec

$\bar{\omega}_r = 11.0$ rad/sec

$\sigma_r = 9.0$ rad/sec

From Fig (3) it can be seen that the simulated results do not fall off fast at higher frequency. This is in support of the conclusions made by Fishbien. Therefore it can be inferred that the simulated results are in agreement with the experimental results.

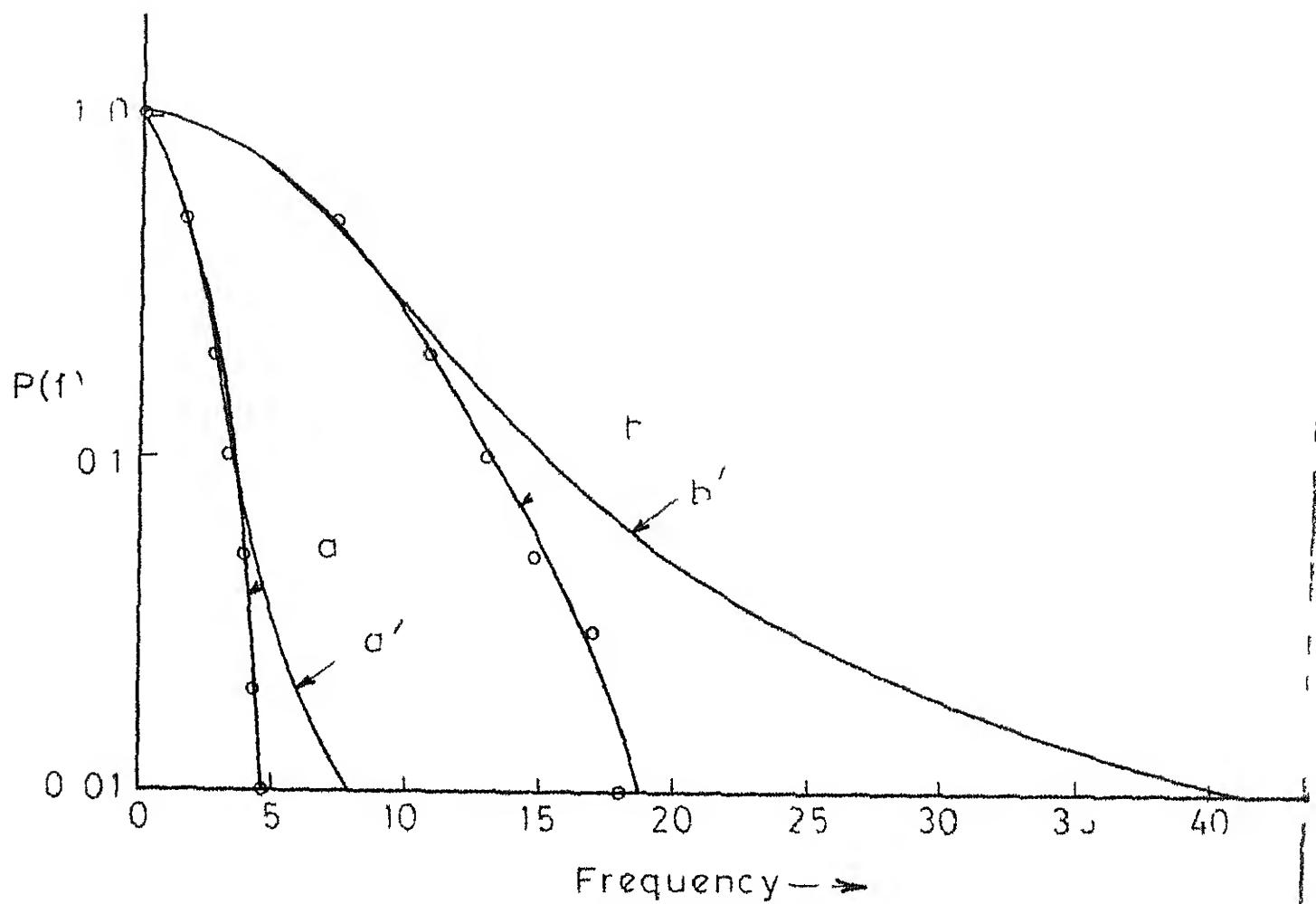


Fig 3 Frequency spectra of various types of complex targets at 1 GHz

EXAMPLE 2

In Fig (4) curve a illustrates the experimental results reported in [9]. It shows the spectra for the echo of chaff cut for $\lambda_c = 10$ cm as measured on $\lambda_c = 9.2$ cm. The data were taken 3 min after the chaff was dispersed from a low moving blimp. The result of simulation is shown by the curve b and the value of parameter in simulation are

$$\bar{\omega}_d = 15.5 \text{ rad/sec}$$

$$\sigma_d = 16.0 \text{ rad/sec}$$

$$\bar{\omega}_r = 15.0 \text{ rad/sec}$$

$$\sigma_r = 2.0 \text{ rad/sec}$$

EXAMPLE 3

Simulated result of the model suggested by J L Wong I S Reed et al are shown by curve - b in Fig (5). Curve - a represents the experimental results and curve c illustrates the simulated results using the present model. In [1] the positive and negative velocities of the scatterers were not considered. From the Fig (2) it can be seen that the curve c is closer to the curve a than the curve - b.

In all the above three examples it was seen that the results simulated using the model suggested in this work are closer to the experimental result obtained than the

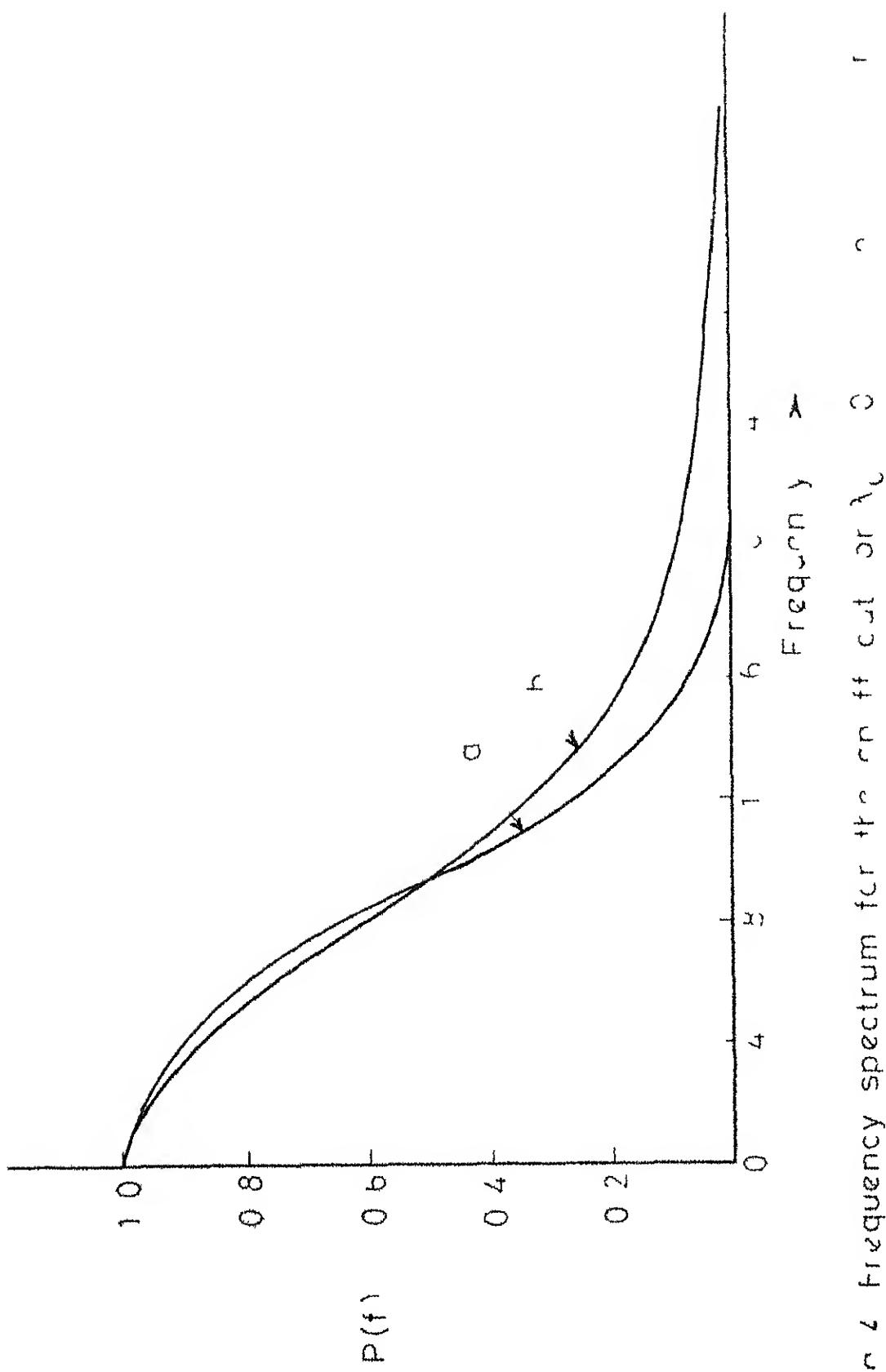
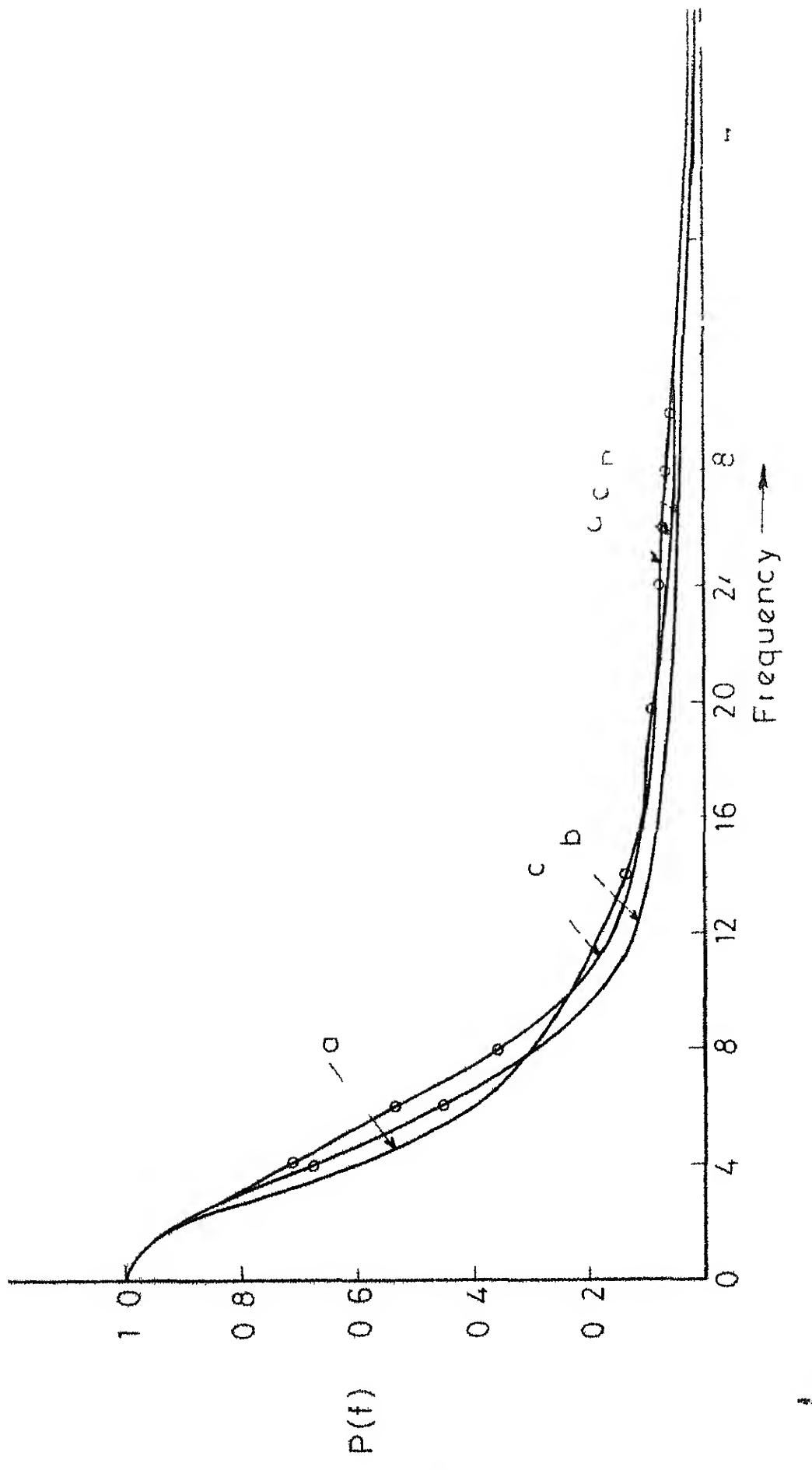


Fig. 5 Function of frequency spectrum for ground clutter in terrain at wind speed 50 mph



previous models. The simulation was done by trial and error method. Better results can be obtained by using gradient method or minimum mean square estimate.

CHAPTER III

FLUCTUATING ELLIPSOIDAL SCATTERER MODEL

In Chapter 2 an expression for the scattering function was derived for an ensemble of random number of point scatterers whose returns follow a non homogeneous Poisson distribution. In this chapter an attempt is made to derive the scattering function for an extended clutter target which following the model suggested by the work of J W. Wright [2] is treated as a finite collection of suitably located ellipsoidal scatterers with varying cross sections. A detailed description of the model along with the derivation of the scattering function is given in Section 3.1. Section 3.2 contains the calculation of the scattering function for a special case.

3.1.1 DESCRIPTION OF THE MODEL

As in [2] the illuminated volume of the scatterers is divided into a finite number of ellipsoids. Each ellipsoid is associated with a suitable modulating function to take care of the irregularities in shapes and also to account for the shadowing effects. A point on the surface of an ellipsoid which has the direction cosines same as those of the line of sight and which is in the same quadrant as the line of sight representative is chosen as the point of the ellipsoid. The radar cross section of the representative point (henceforth referred to as specular point) is equal to the RCS of the ellipsoid calculated at the specular point. The advantage of choosing

LIBRARY
66955

ellipsoids and representing each of them by a single point is that a large number of scatterers in the ellipsoid are represented by a single point thus making the effective number of scatterers in the illuminated volume a finite number

Three coordinate systems are chosen to describe the ellipsoids. First is the radar fixed coordinate system (r c s) second is the target fixed coordinate system (t c s) which is parallel to the first and the third is the local coordinate system (l c s). The transformation between the first two coordinate systems is linear and therefore the aspect angles of one with respect to the other are equal. The l c s is the ellipsoid's own coordinate system with its centre coinciding with that of the ellipsoid. The axes of the l c s are the axes of the ellipsoid.

The vectorial representation of the three coordinate systems (Fig. (6)) is as follows

$$\begin{aligned}
 \overrightarrow{O_r t} &= i_r x + j_r y + k_r z \\
 \overrightarrow{O_t P} &= i_t x + j_t y + k_t z \\
 \overrightarrow{O_l P} &= i_l x_l + j_l y_l + k_l z_l \quad (3.1.1) \\
 \overrightarrow{O_r O_t} &= i_r x_o + j_r y_o + k_r z_o \\
 \overrightarrow{O_t O_l} &= i_t x_{ti} + j_t y_{ti} + k_t z_{ti}
 \end{aligned}$$

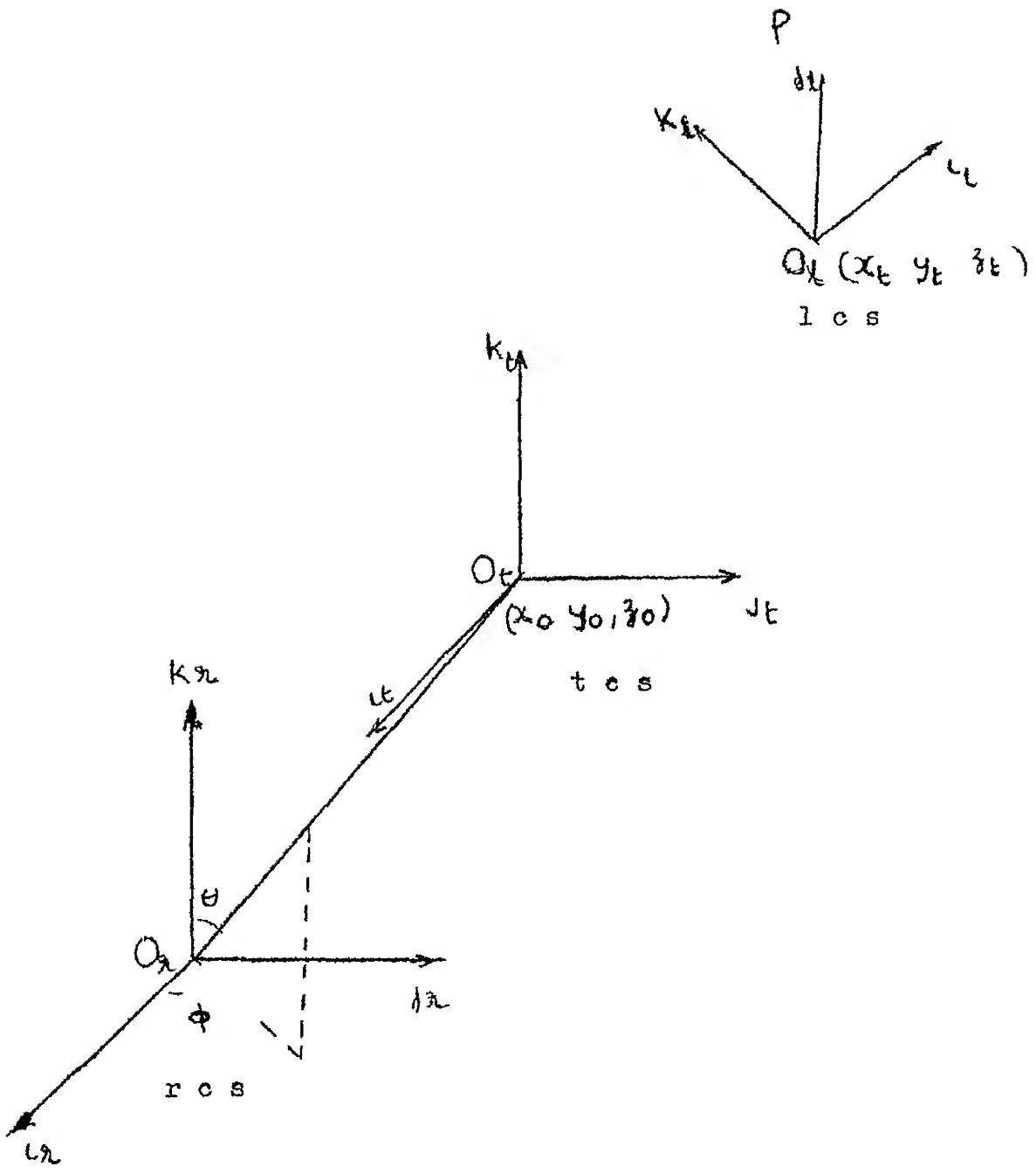


Fig 6

The relationship between the t c s and r c s is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x & x_0 \\ y & y_0 \\ z & z_0 \end{bmatrix} \quad (3 \ 1 \ 2)$$

To allow for arbitrary orientations of the ellipsoids we consider translational and rotational transformations of the t c s to obtain the l c s. After an initial translational transformation to the centre of the ellipsoid two rotational transformations are followed by such that the axes of the l c s coincide with the axes of the ellipsoid. In the rotational transformation the axes are rotated initially by an angle ϕ_t about the z axis and then by an angle θ_t about the new y axis. The coordinates of a point in the target fixed coordinate system and the local coordinate system are related by

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = [T_1] \begin{bmatrix} x & x_{t1} \\ y & y_{t1} \\ z & z_{t1} \end{bmatrix} \quad (3 \ 1 \ 3)$$

where

$$[T_1] = \begin{bmatrix} \cos \phi_{t1} \cos \theta_{t1} & \sin \phi_{t1} & \cos \theta_{t1} & \sin \theta_{t1} \\ \sin \phi_{t1} & \cos \phi_{t1} & 0 & 0 \\ \cos \phi_{t1} \sin \theta_{t1} & \sin \phi_{t1} & \sin \theta_{t1} & \cos \theta_{t1} \end{bmatrix} \quad (3 \ 1 \ 4)$$

where ϕ_{t1} and θ_{t1} are right handed angles and the subscript 1 indicates the 1th l c s

LIST OF SYMBOLS

r c s	radar fixed coordinate system
t c s	- target fixed coordinate system
l c s	local coordinate system
(x_0, y_0, z_0)	- origin of the t c s in the r c s
(x_{t1}, y_{t1}, z_{t1})	origin of the l c s in the t c s
(x, y, z)	coordinates of a point in the r c s
(x, y, z)	coordinates of a point in the t c s
(x_1, y_1, z_1)	coordinates of the specular point of the 1th ellipsoid in the l c s
θ, ϕ	- aspect angles of the r c s in the t c s
θ_{li}, ϕ_{li}	- aspect angles of the centre of the 1th l c s in the t c s
θ_1, ϕ_1	aspect angles of the 1th specular point in the 1th l c s
θ_{t1}, ϕ_{t1}	- describe the transformation between l c s and t c s
R_0	distance between the centre of the r c s and the centre of the t c s
a_1, b_1, c_1	- semi axes of the 1th ellipsoid
M_1	modulating function of the 1th ellipsoid
R_{11}, R_{21}	Radii of curvature of the 1th ellipsoid at its specular point
V_1	voltage reflection coefficient of the 1th scatterer
σ_1	RCS of the 1th scatterer
v_1	linear velocity of the 1th scatterer

τ_i delay of the i th scatterer

λ_c transmitted wave length

c velocity of light

NOTE i th specular/ is referred to as the i th scatterer

3.1.2 DETERMINATION OF THE VOLTAGE REFLECTION COEFFICIENT

The RCS of the i th ellipsoid is given by (2)

$$S_i = M_i \pi R_{1i} R_{2i}$$

where M_i is the modulating function of the i th ellipsoid
 R_{1i} and R_{2i} are the radii of curvature of the i th ellipsoid
at its specular point calculate the coordinates of the
specular point and the product of the radii of curvature as
below

Let

$$F_i(x, y, z) = \frac{x^2}{a_i^2} + \frac{y^2}{b_i^2} + \frac{z^2}{c_i^2} - 1 = 0 \quad (3.1.6)$$

be the equation of the i th ellipsoid with semi axes a_i , b_i
and c_i . The normal to the ellipsoid at (x_i, y_i, z_i) is given
by

$$\vec{N} = \frac{2x_i}{a_i^2} \hat{i}_1 + \frac{2y_i}{b_i^2} \hat{j}_1 + \frac{2z_i}{c_i^2} \hat{k}_1 \quad (3.1.7)$$

From the above the direction cosines of the normal are

$$\begin{aligned}\cos \delta_x &= \frac{x_1}{\frac{a_1^2}{2} r} \\ \cos \delta_y &= \frac{y_1}{\frac{b_1^2}{2} r} \\ \cos \delta_z &= \frac{z_1}{\frac{c_1^2}{2} r}\end{aligned}\tag{3.1.8}$$

where

$$r = \sqrt{\frac{x^2}{\frac{a_1^2}{4}} + \frac{y^2}{\frac{b_1^2}{4}} + \frac{z^2}{\frac{c_1^2}{4}}}$$

The direction cosines of the line of sight in the target coordinate system are

$$\begin{aligned}\cos \delta_x &= \sin \theta \cos \phi \\ \cos \delta_y &= \sin \theta \sin \phi \\ \cos \delta_z &= \cos \theta\end{aligned}\tag{3.1.9}$$

But the direction cosines of the line of sight in 1 c are

$$\begin{aligned}\cos \delta_x &= \sin \theta_1 \cos \phi_1 \\ \cos \delta_y &= \sin \theta_1 \sin \phi_1 \\ \cos \delta_z &= \cos \theta_1\end{aligned}\tag{3.1.10}$$

where

$$\begin{bmatrix} \sin \theta_1 & \cos \phi_1 \\ \sin \theta_1 & \sin \phi_1 \\ \cos \theta_1 \end{bmatrix} = [T_1] \begin{bmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{bmatrix} \quad (3.1.11)$$

The normal to the ellipsoid at the specular point should have the same direction cosines as the line of sight and should be in the same quadrant as the line of sight. From the above equations the parametric angular coordinate u_s and v_s of the specular point are obtained as

$$v_{s1} = \tan^{-1} \left\{ \frac{b_1}{a_1} \tan \phi_1 \right\} \quad (3.1.12)$$

$$u_{s1} = \tan^{-1} \left\{ \frac{1}{c_1} (\tan \theta_1) \sqrt{a_1^2 \cos^2 \phi_1 + b_1^2 \sin^2 \phi_1} \right\}$$

$R_{11} R_{21}$ as given in [2] is

$$R_{11} R_{21} = \frac{\left(\frac{\partial F_1}{\partial x_1} \right)^2 + \left(\frac{\partial F_1}{\partial y_1} \right)^2 + \left(\frac{\partial F_1}{\partial z_1} \right)^2}{\Delta} \quad (3.1.13)$$

where i th sp cul r point

$$= \begin{vmatrix} \frac{\partial^2 F_1}{\partial x^2} & \frac{\partial^2 F_1}{\partial x \partial y} & \frac{\partial^2 F_1}{\partial x \partial z} & \frac{\partial F_1}{\partial x} \\ \frac{\partial^2 F_1}{\partial x \partial y} & \frac{\partial^2 F_1}{\partial y^2} & \frac{\partial^2 F_1}{\partial y \partial z} & \frac{\partial F_1}{\partial y} \\ \frac{\partial^2 F_1}{\partial x \partial z} & \frac{\partial^2 F_1}{\partial y \partial z} & \frac{\partial^2 F_1}{\partial z^2} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} & 0 \end{vmatrix}$$

$$\text{Hence } R_{11} R_{21} = a_1^2 b_1^2 c_1^2 \left(\frac{x_1^2}{a_1^4} + \frac{y_1^2}{b_1^4} + \frac{z_1^2}{c_1^4} \right)^2 \quad (3.1.14)$$

where (x_1, y_1, z_1) are the coordinates of the specular point in the local and they are determined by equating (3.1.8) and (3.1.10)

$$\begin{aligned} x_1 &= a_1 \sin u_{si} \cos v_{si} = \frac{a_1^2}{P_1^2} \sin \theta_1 \cos \phi_1 \\ y_1 &= b_1 \sin u_{si} \sin v_{si} = \frac{b_1^2}{P_1^2} \sin \theta_1 \sin \phi_1 \quad (3.1.15) \\ z_1 &= c_1 \cos u_{si} = \frac{c_1^2}{P_1^2} \cos \theta_1 \end{aligned}$$

$$\text{where } P_1^2 = a_1^2 \sin^2 \theta_1 \cos^2 \phi_1 + b_1^2 \sin^2 \theta_1 \sin^2 \phi_1 + c_1^2 \cos^2 \theta_1 \quad (3.1.1)$$

From the above it can be seen that θ_1 and ϕ_1 are also the aspect angles of the i th specular point

Since the RCS of the i th ellipsoid is determined from (3.1.9), (3.1.14) and (3.1.15)

$$S_i = M_i \pi R_{11} R_{21} = \frac{M_i \pi a_1^2 b_1^2 c_1^2}{P_1^4} \quad (3.1.17)$$

To derive the scattering function it is also necessary to determine the phase of the scattered electromagnetic field at the receiver. The relative phase difference between the reflected electromagnetic waves of two scatterers is given by

$$\alpha_i = \frac{4\pi}{\lambda_c} \Delta R_i \quad (3.1.18)$$

where ΔR_i is the difference in range

The phase of the electric field scattered by the i th scatterer can be determined by using the target coordinate centre as the reference point. Using this

$$\Delta R_i = R_o - R_i = x^1 \sin \theta \cos \phi \quad y^1 \sin \theta \sin \phi \quad z^1 \cos \theta \quad (3.1.19)$$

where R_o is the distance between the radar and the target coordinate systems. R_i is the distance between the i th specular point and the rcs and $x^1 \quad y^1 \quad z^1$ denote the coordinates of the specular point in rcs

From (3.1.18) and (3.1.19)

$$\begin{aligned} \alpha_i &= \frac{4\pi}{\lambda_c} [-x^1 \sin \theta \cos \phi \quad y^1 \sin \theta \sin \phi \quad z^1 \cos \theta] \\ &= \frac{4\pi}{\lambda_c} \begin{bmatrix} x^1 \\ y^1 \\ z^1 \end{bmatrix}^t \begin{bmatrix} \sin \theta & \cos \phi \\ \sin \theta & \sin \phi \\ \cos \theta \end{bmatrix} \end{aligned}$$

From (3.1.3)

$$\begin{bmatrix} x^1 \\ y^1 \\ z^1 \end{bmatrix} = [T_i]^{-1} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \begin{bmatrix} x_{ti} \\ y_{ti} \\ z_{ti} \end{bmatrix}$$

therefore

$$\alpha_i = \frac{4\pi}{\lambda_c} \left\{ \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^t [T_i^{-1}]^t + \begin{bmatrix} x_{ti} \\ y_{ti} \\ z_{ti} \end{bmatrix}^t \right\} \begin{bmatrix} \sin \theta & \cos \phi \\ \sin \theta & \sin \phi \\ \cos \theta \end{bmatrix}$$

Since $[T]$ is an orthogonal matrix

$$[T_1^{-1}]^t = [T_1] \quad \forall \quad 1$$

Hence

$$\alpha_1 = \frac{4\pi}{\lambda_c} \left\{ \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}^t [T_1] \begin{bmatrix} \sin\theta \cos\phi \\ \sin\theta \sin\phi \\ \cos\theta \end{bmatrix} + \begin{bmatrix} x_{t1} \\ y_{t1} \\ z_{t1} \end{bmatrix}^t \begin{bmatrix} \sin\theta \cos\phi \\ \sin\theta \sin\phi \\ \cos\theta \end{bmatrix} \right\}$$

From (3.1.11)

$$\begin{aligned} \alpha_1 &= \frac{4\pi}{\lambda_c} \left\{ \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}^t \begin{bmatrix} \sin\theta_1 \cos\phi_1 \\ \sin\theta_1 \sin\phi_1 \\ \cos\theta_1 \end{bmatrix} + \begin{bmatrix} x_{t1} \\ y_{t1} \\ z_{t1} \end{bmatrix}^t \begin{bmatrix} \sin\theta \cos\phi \\ \sin\theta \sin\phi \\ \cos\theta \end{bmatrix} \right\} \\ &= \frac{4\pi}{\lambda_c} [(x_1 \sin\theta_1 \cos\phi_1 + y_1 \sin\theta_1 \sin\phi_1 + z_1 \cos\theta_1) + \\ &\quad (x_{t1} \sin\theta \cos\phi + y_{t1} \sin\theta \sin\phi + z_{t1} \cos\theta)] \end{aligned}$$

Substituting for $x_1 \ y_1 \ z_1$ from (3.1.15)

$$\begin{aligned} \alpha_1 &= \frac{-4\pi}{\lambda_c} \left[\frac{1}{P_1} (a_1^2 \sin^2\theta_1 \cos\phi_1 + b_1^2 \sin^2\theta_1 \sin^2\phi_1 + c_1^2 \cos^2\theta_1) \right. \\ &\quad \left. + (x_{t1} \sin\theta \cos\phi + y_{t1} \sin\theta \sin\phi + z_{t1} \cos\theta) \right] \end{aligned}$$

From (3.1.16)

$$\alpha_1 = \frac{4\pi}{\lambda_c} [P_1 + (x_{t1} \sin\theta \cos\phi + y_{t1} \sin\theta \sin\phi + z_{t1} \cos\theta)] \quad (3.1.17)$$

Therefore the reflected field from the i th scatterer has a phase dependent $e^{j\alpha_1}$ relative to the target coordinate system. The voltage reflection coefficient of the i th scatterer can be

calculated using the R S of the same. The relationship between the above two is given by

$$|\bar{v}_1^2| = \frac{c^2 \lambda^2}{(4\pi)^3 R_1^4} |\bar{\sigma}_1| \quad (3.1.20)$$

From (3.1.17) σ_1 is

$$\sigma_1 = \frac{M_1 \pi a_1^2 b_1^2 c_1^2}{p_1^4} \quad (3.1.21)$$

therefore

$$|v_1| = \frac{G \lambda c M_1}{(4\pi)^{3/2} R_1^2} \sqrt{\frac{a_1^2 b_1^2 c_1^2}{p_1^4}} \quad (3.1.22)$$

By taking into account the phase term and assuming that the scatterers are very far from the radar such that $R_1^2 = R_J^2 - R_o^2$, \bar{v}_1 may then be expressed as

$$\bar{v}_1 = \frac{G \lambda c}{8\pi R_o^2} M_1 \frac{a_1 b_1 c_1}{p_1^2} e^{j(a_1 + \beta)} \quad (3.1.23)$$

in which we have also associated a random phase shift β with the reflection process

3.1.3 DERIVATION OF THE SCATTERING FUNCTION

The scatterers under the effects of wind forces have linear and rotational movements. If the wind velocities are large the movement of the scatterers causes variations in the echo signal which can not be neglected. In the present model to account for the random motions of the scatterers it is assumed that the ellipsoids have random fluctuations about

their mean positions and linear velocities v_1 along the line of sight. This implies that the local coordinate system has random fluctuations and consequently θ_t and ϕ_t are random processes. The size of the ellipsoid is also assumed to change randomly thus making the semi axes a , b and c random processes. Then

$$\tilde{v}_1(t) = \frac{G \lambda_0}{8 \pi R_0^2} M_1 \frac{a_1(t) b_1(t) c_1(t)}{P_1^2(t)} e^{j[\alpha_1(t) + \beta]} \quad (3.1.24)$$

Let $\tilde{f}(t)$ be the complex envelope of the transmitted signal then the complex envelope of the echo signal from the i th scatterer is given by

$$\tilde{s}_{ri}(t) = \tilde{v}_1(t - \frac{\lambda_i(t)}{2}) f(t - \lambda_1(t)) e^{j\omega_c \lambda_1(t) + j\beta} \quad (3.1.25)$$

where ω_c is the carrier frequency and $\lambda_i(t)$ is the round trip delay

$\lambda_1(t)$ as given in (2.2.36) is

$$\lambda_1(t) = \lambda_1 - 2 \frac{v_1 t}{c} \quad (3.1.26)$$

where v_1 is positive if the scatterers are moving towards the radar

Substituting

$$\tilde{x}_1(t) \triangleq \tilde{v}_1(t - \frac{\lambda_i(t)}{2}) e^{j\omega_c \lambda_1(t) + j\beta} \quad (3.1.27)$$

eqn (3.1.25) reduces to

$$s_{ri}(t) = \tilde{x}_1(t) \tilde{f}(t - \lambda_1(t)) \quad (3.1.28)$$

Assuming that random phase β is uniformly distributed in $(0, 2\pi)$ it can be seen that the process $x_1(t)$ has zero mean

Assuming that there are K scatterers whose echoes arrive with the same delay λ the total received signal with the delay λ is

$$\tilde{s}_r(t - \lambda) = \sum_{i=1}^K \tilde{x}_i(t) f(t - \lambda_i(t)) \quad (3.1.29)$$

$$\text{where } \lambda_i(t) = \lambda_1 - \frac{2v}{c} t$$

and

$$\lambda_1 = \lambda - \frac{2v}{c} t$$

Then the autocorrelation of the received signal with the delay λ is

$$\begin{aligned} \tilde{R}_{S_r}(t_\alpha - t_\beta, \lambda) &= E\{S_r(t_\alpha - \lambda) \tilde{s}_r(t_\beta - \lambda)\} \\ &= E\left\{\sum_{i=1}^K \sum_{j=1}^K x_i(t_\alpha) x_j^*(t_\beta) f(t_\alpha - \lambda_i(t_\alpha)) f^*(t_\beta - \lambda_i(t_\beta))\right\} \quad (3.1.30) \end{aligned}$$

From (3.1.30) the autocorrelation function of the process x with the delay λ is

$$\tilde{R}_x(t_\alpha - t_\beta, \lambda) = E\left\{\sum_{i=1}^K \sum_{j=1}^K x_i(t_\alpha) x_j^*(t_\beta)\right\} \quad (3.1.31)$$

Assuming that the scatterers reflect energy independently and since the $x_i(t)$ s are zero mean random processes the above reduces to

$$\tilde{R}_x(t_\alpha - t_\beta, \lambda) = \sum_{i=1}^K E\{x_i(t_\alpha) x_i^*(t_\beta)\} \quad (3.1.32)$$

, $x_1(t)$ is stationary

$$R_x(\tau - \lambda) = \sum_{i=1}^I R_{x1}(\tau) \quad (3.1.33)$$

Then the normalized correlation function of the process x_1 is given by

$$\epsilon(\tau - \lambda) = \frac{R_x(\tau - \lambda)}{R_x(0)} \quad (3.1.34)$$

From (3.1.34) the scattering function $(f \lambda)$ is given by

$$S(f \lambda) = \int_{-\infty}^{\infty} g(\tau - \lambda) e^{-j2\pi f \tau} d\tau \quad (3.1.35)$$

3.2 EXAMPLE

In this section to illustrate the procedure described in Section 3.1 with more detail a special case of the model is considered and an expression for the scattering function is evaluated for the same.

Here it is assumed that the ellipsoids are fluctuating in size along the y axis but their size is constant along the x and z axes, i.e. $b_1(t)$ is a random process and $a_1(t)$, $c_1(t)$ are constants.

$$\text{Let } a_1(t) = c_1(t) = a_1 \quad \forall i$$

and $M_1 = 1 \quad (3.2.1)$

It is also assumed that $\phi_{t1} = 0 \quad \forall i$

$$\text{and } \phi = 0$$

i.e. the y axis of the t c's and the y axis of the l c's are parallel. With these assumptions the transformation matrix $[T_1]$ is as given below

$$[T_1] = \begin{bmatrix} \cos \theta_{t_1}(t) & 0 & \sin \theta_{t_1}(t) \\ 0 & 1 & 0 \\ \sin \theta_{t_1}(t) & 0 & \cos \theta_{t_1}(t) \end{bmatrix} \quad (3 \ 2 \ 2)$$

From (3 1 11)

$$\begin{bmatrix} \sin \theta_1(t) \cos \phi_1(t) \\ \sin \theta_1(t) \sin \phi_1(t) \\ \cos \theta_1(t) \end{bmatrix} = \begin{bmatrix} \cos \theta_{t_1}(t) & 0 & \sin \theta_{t_1}(t) \\ 0 & 1 & 0 \\ \sin \theta_{t_1}(t) & 0 & \cos \theta_{t_1}(t) \end{bmatrix} \begin{bmatrix} \quad \\ \quad \\ t \end{bmatrix} \quad (3 \ 2 \ 3)$$

Hence

$$\begin{aligned} \sin \theta_1(t) \cos \phi_1(t) &= \sin(\theta_{t_1}(t) + \theta) \\ \sin \theta_1(t) \sin \phi_1(t) &= 0 \\ \cos \theta_1(t) &= \cos(\theta_{t_1}(t) + \theta) \end{aligned} \quad (3 \ 2 \ 4)$$

From (3 1 16) and (3 2 4)

$$P_1(t) = a \quad (3 \ 2 \ 5)$$

Then the coordinates of the specular point are

$$\begin{aligned}
 x_1 &= \frac{a_1^2}{P_1^2(t)} \sin \theta_1(t) \cos \phi_1(t) & a_1 \sin(\theta_{t1}(t) + \theta) \\
 y_1 &= \frac{b_1^2}{P_1^2(t)} \sin \theta_1(t) \sin \phi_1(t) & 0 \\
 z_1 &= \frac{c_1^2}{P_1^2(t)} \cos \theta_1(t) & a_1 \cos(\theta_{t1}(t) + \theta)
 \end{aligned} \tag{326}$$

From the above the distance from the centre of the l c s to the specular point is

$$R_1 = \sqrt{a_1^2 + z_1^2} \tag{327}$$

From (327) it can be inferred that the fluctuations of the ellipsoids about y axis alone do not have any effect on R_1

Rewriting (3119a)

$$\begin{aligned}
 \alpha_1(t) &= \frac{4\pi}{\lambda_c} [P_1(t) + x_{t1}(t) \sin \theta \cos \phi + \\
 &\quad y_{t1}(t) \sin \theta \sin \phi + z_{t1}(t) \cos \theta]
 \end{aligned} \tag{328}$$

Substituting for $P_1(t)$ from (321) and (325)

$$\alpha_1(t) = \frac{4\pi}{\lambda_c} [a_1 + x_{t1}(t) \sin \theta \cos \phi + z_{t1}(t) \cos \theta] \tag{329}$$

In the above $(x_{t1} \ y_{t1} \ z_{t1})$ is the centre of the l c s. This does not change with fluctuations in θ_{t1} . Therefore the phases of the returns from the specular point are unaffected by the fluctuations in θ_{t1} . $x_{t1} \ z_{t1}$ can be written as

$$\begin{aligned}
 x_{t1}(t) &= R_{t1}(t) \sin \theta_{11} \cos \phi_{11} \\
 z_{t1}(t) &= R_{t1}(t) \cos \theta_{11}
 \end{aligned} \tag{3210}$$

where $R_{ti}(t)$ is the distance between the centres of l c s and t c s and is given by

$$R_{ti}(t) = \sqrt{x_{ti}^2(t) + y_{ti}^2(t) + z_{ti}^2(t)}$$

and θ_{li} ϕ_{li} are the aspect angles of the centre of the l c s in t c s

Substituting (3 2 10) in (3 2 9)

$$\begin{aligned} a_1(t) = & \frac{4\pi}{\lambda_c} [a_1 + R_{ti}(t) \sin \theta_{li} \cos \phi_{li} \sin \theta + R_{ti}(t) \\ & + R_{ti}(t) \cos \theta_{li} \cos \theta] \end{aligned} \quad (3 2 11)$$

Since the ellipsoids are assumed to have linear velocities along the line of sight

$$R_{ti}(t) = R_{ti}(0) + v_1 t$$

Therefore

$$\begin{aligned} \tilde{x}_1(t) = & \frac{G \lambda}{8 \pi R_0^2} c b_1(t - \frac{\lambda_1(t)}{2}) \exp [j \frac{4\pi}{\lambda_c} \{ a_1 + R_{ti}(t - \frac{\lambda_1(t)}{2}) \\ & (\sin \theta \sin \theta_{li} \cos \phi_{li} + \cos \theta \cos \theta_{li}) \} - j \omega_c \lambda_1(t) + j \beta] \end{aligned}$$

Assuming that $v_1 \ll c$,

and substituting $c_1 = \sin \theta \sin \theta_{li} \cos \phi_{li} + \cos \theta \cos \theta_{li}$

$$\begin{aligned} \tilde{x}_1(t) = & \frac{G \lambda}{8 \pi R_0^2} b_1(t - \frac{\lambda_1(t)}{2}) \exp [- \frac{j 4\pi}{\lambda_c} (a_1 + (R_{ti}(0) + \frac{v_1 \lambda_1}{2}) c_1) \\ & - j \omega_c \lambda_1 + j \beta + \frac{j 4\pi}{\lambda_c} c_1 v_1 t + j \omega_c \frac{v_1 \lambda_1}{c} t] \end{aligned} \quad (3 2 12)$$

β is the random phase incurred in the reflection process. Assuming β to be uniform in $(0 \quad 2\pi)$ it can be seen that the process $x_1(t)$ has zero mean

The autocorrelation of the process $x_1(t)$ is given by

$$\begin{aligned}
 R_{x_1}(t_\alpha - t_\beta) &= E\{x_1(t_\alpha)x_1^*(t_\beta)\} \\
 &= \frac{G^2 \lambda^2}{8\pi R_0^2} E\{b_1(t_\alpha - \frac{\lambda_1(t_\alpha)}{2})b_1^*(t_\beta - \frac{\lambda_1(t_\beta)}{2})\} \\
 &= E\{\exp[i\frac{4\pi}{\lambda}c_1 v_1(t_\alpha - t_\beta) + i\omega_c \frac{1}{c}(t_\alpha - t_\beta)]\}
 \end{aligned} \tag{3.2.13}$$

where $\tilde{b}_1(t)$ is independent of the rest of the random quantities in the above equation

Assuming $b_1(t)$ to be stationary

$$E\{\tilde{b}_1(t_\alpha - \frac{\lambda_1(t_\alpha)}{2})\tilde{b}_1^*(t_\beta - \frac{\lambda_1(t_\beta)}{2})\} = R_{b_1}(\tau) \tag{3.2.14}$$

where $\tau = t_\alpha - t_\beta$

$$\text{Substituting } c_2 = \frac{G^2 \lambda^2}{8\pi R_0^2} \text{ and } \omega_{d_1} = \frac{2\omega_c \lambda_1}{c}(1+c_1) \tag{3.2.15}$$

From the above (3.2.14) reduces to

$$R_{x_1}(t_\alpha - t_\beta) = c_2 R_{b_1}(\tau) E\{\exp[i\omega_{d_1}(-\tau)]\} \tag{3.2.16}$$

The scatterers can move towards or away from the radar along the line of sight with equal probability with their velocity distributions centred about $\pm \bar{w}_d$

Therefore from (2 2 42)

$$E [e^{j \omega_{d1}(\tau)}] = \cos(\bar{\omega}_{d1} \tau) \phi_{\omega_{d1}}(\tau) \quad (3 2 17)$$

Therefore

$$R_{x1}(t_\alpha, t_\beta) = c_2 R_{b1}(\tau) \phi_{\omega_{d1}}(\tau) \cos(\bar{\omega}_{d1} \tau) \quad (3 2 18)$$

From the above it can be seen that $x_1(t)$ is a stationary process

$$R_{\hat{x}1}(\tau) = c_2 R_{b1}(\tau) \phi_{\omega_{d1}}(\tau) \cos(\bar{\omega}_{d1} \tau) \quad (3 2 19)$$

From (3 1 33)

$$R_x(\tau, \lambda) = \sum_{l=1}^K R_{\hat{x}1}(\tau) - \sum_{l=1}^K c_2 R_{b1}(\tau) \phi_{\omega_{d1}}(\tau) \cos(\bar{\omega}_{d1} \tau) \quad (3 2 20)$$

The normalized correlation function of the process x is given by

$$g(\tau, \lambda) = \frac{R_x(\tau, \lambda)}{R_x(0, \lambda)} \quad (3 2 21)$$

From (3 2 20) and (3 2 21)

$$g(\tau, \lambda) = \sum_{l=1}^K R_{b1}(\tau) \phi_{\omega_{d1}}(\tau) \cos(\bar{\omega}_{d1} \tau) \quad (3 2 22)$$

and the scattering function is

$$S(f, \lambda) = \int_{-\infty}^{\infty} g(\tau, \lambda) e^{j 2\pi f \tau} d\tau \quad (3 2 23)$$

Thus the scattering function can be calculated for other models with a procedure similar to the one described above. For the scattering function to exist the assumptions on the random nature of various parameters should be made appropriately.

CHAPTER IV

CONCLUSIONS

In this chapter the results obtained in Chapter 2 and Chapter 3 are summarised and some suggestions for further work in this area are made.

This thesis has attempted to provide a statistical characterization of clutter in terms of the scattering function for two models of clutter returns. Chapter 2 contains the first model in which the scatterers are modelled as random rotating dipoles with an overall drift velocity and differential velocities. An expression for the scattering function is derived by calculating the voltage reflection coefficients of an ensemble of dipoles with a non homogeneous Poisson distribution. An attempt has been made to generate a few scattering function which compare favourably with some of the reported results.

Chapter 3 consists of a model in which the clutter targets are assumed to be a collection of ellipsoidal scatterers with varying cross sections. The ellipsoids are assumed to fluctuate in size and also about their mean positions. As an illustration an expression for the scattering function is obtained for a relatively simple geometry and aspect angle fluctuations.

To evaluate the adequacy of the models for clutter considered in this the is it is necessary to generate scattering functions by extensive simulation for a wide class of parametrically described probability distributions and compare them with experimentally obtained scattering functions on a more systematic basis than has been attempted here

As clutter statistics are well known to exhibit non stationary statistics it is also desirable to examine whether these models can be used to generate such statistics

BIBLIOGRAPHY

- 1 J L Wong I S Reed and Z A Kaprielian A Model for the Radar Echo from a Random Collection of Rotating Dipole Scatterers IEE Trans Aerospace Vol AES 3 No 2 March 1967 pp 171 178
- 2 James W Wright et al On The Statistical Modelling of Radar Targets Army Missile Command Redstone Arsenal Alabama Nov 72 AD 753936 NTIS Report
- 3 H L Vantrees Optimum Signal Design and Processing for Reverberation limited Environments IEE Trans Military Electronics MIL 9 July 1965 pp 212 229
- 4 D C Childers and I S Reed A Model for the Power Spectrum of return d echoes from random collection of moving scatterers Univ of Southern California School of Engg Rept 102 Dec 1963
- 5 H L Vantrees Detection Estimation and Modulation Theory Part III John Wiley & Sons Inc 1971
- 6 Maurice W Long Radar reflectivity of land and sea Lexington Books 1975
- 7 J W Crispin and K E Siegel Methods of Radar Cross section Analysis Academic Press 1968
- 8 D L Kerr Propagation of Short Radio Waves MIT Rad Lab Series Vol 13 N Y McGraw Hill 1950
- 9 J L Lawson and G E Uhlenbeck Threshold Signals MIT Rad Lab Series Vol 24 N Y McGraw Hill 1950

- 10 Collins and Zucker Antenna Theory Part II McGraw-Hill 1969
- 11 Corson and Lorrain Introduction to Electromagnetic Fields and Waves W H Freeman and Co 1962
- 12 G T Ruck D E Barrick W D Stuart and C K Krichbaum Radar Cross section Handbook Plenum Press N Y 1970
- 13 J F Siegert On the fluctuation in signals returned by many independent scatterers M I T Rad Lab Cambridge Mass Rept 465 Nov 12 1943
- 14 E J Kelly and L C Lerner A mathematical model for the radar echo from a random collection of scatterers M I T Lincoln Lab Lexington Mass Rept 123 June 15 1956
- 15 M I Skolnik Introduction to Radar Systems McGraw-Hill Kogakusha Ltd 1926
- 16 A Papoulis Probability Random variables and stochastic processes McGraw Hill Kogakusha Ltd 1965
- 17 E Parzen Stochastic Processes Holden Day Inc 1962
- 18 S Karlin and H M Taylor A first course in stochastic processes Academic Press Inc 1975
- 19 F L Nathanson and J P Reilly Clutter statistics which affect radar performance analysis Supplement to the IEEE Trans on Aerospace and electronic systems Vol ALS 3 No 6 Nov 1967
- 20 R L Mitchell Radar signal simulation Artech House Inc 1976

- 21 Stevenson A F Electromagnetic scattering by an ellipsoid in the third approximation J Appl Phys 24 1143 1953
- 22 J T Evans and T Hagfors Radar Astronomy M I T Lincoln Lab McGraw Hill 1968
- 23 David K Barton Radars Volume V Artech House 1975

A 66955

Date Slip **A 66955**

Thi b k l t b t d nth
J mp d

--

CD 6729

EE-1980-M-RAT-STU